NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3757

TORSIONAL INSTABILITY OF HINGED FLANGES STIFFENED BY LIPS AND BULBS

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SUMMARY

Based on torsional instability theory, buckling charts are presented for determining the critical strain of hinged flanges stiffened by idealized lip and bulb elements.

INTRODUCTION

The buckling of a stiffened flange has been treated by several investigators (refs. 1 to 7). The results of all these investigations are either for sections of specific shape or else in a form which cannot be readily used by the stress analyst.

The primary purpose of the addition of a lip or bulb is to stiffen the free edge of a flange in order to increase the buckling stress of this member. If the lip is too large relative to the flange, it may buckle at a stress less than that required to buckle the stiffened flange. If the lip is too small, torsional buckling of the stiffened flange may occur at a stress less than that at which the flange would act as a plate simply supported along both unloaded edges, for example.

In this report, interest is centered on the torsional buckling behavior of simply supported flanges stiffened by relatively small idealized lip or bulb elements. The lower limit is that of the unstiffened flange and the upper limit considered is that of a stiffened flange which acts as a long simply supported plate. Buckling charts are presented which cover the range between these limiting cases.

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SYMBOLS

A	area, sq in.
b_f	flange width, in.
b_L	lip width, in.
b_w	web width, in.
C	torsion-bending constant, in.^{6}
d	bulb diameter, in.
E	modulus of elasticity, ksi
E_s	secant modulus, ksi
G	shear modulus of elasticity, ksi
I_p	polar moment of inertia, in.^{4}
J	torsion constant, in.^{4}
L	length, in.
t	thickness, in.
x,y	coordinates
ϵ_{cr}	critical strain
λ	half wave length of buckles, in.
ν	Poisson's ratio, 0.3
σ_{cr}	critical stress, ksi

Subscript:

s	lip or bulb

THEORY

The determination of the torsional buckling stress of lip- and bulb-stiffened flanges with simple support along the unloaded edge is based upon the following assumptions:

(1) The idealized configurations shown in figure 1 are used in the analysis.

(2) The stiffened flange rotates about a hinge line at the base of the flange in the torsional buckling mode. As shown in figure 2(a), this mode involves rotation of the stiffened flange about this axis with no distortion of the cross section.

(3) Local buckling of the stiffened flange which involves distortion of the cross section but no rotation, as shown in figure 2(b), is not considered.

(4) The assumptions of the torsion-bending theory discussed in references 4 and 8 are retained in this analysis.

In reference 8, the torsional buckling stress of a stiffened hinged flange may be obtained from the following equation

$$\sigma_{cr} = \frac{GJ}{I_p} + \frac{\pi^2 EC}{I_p \lambda^2} \tag{1}$$

The axis of rotation of the flange is at the origin of the coordinate system. Thus, as a close approximation

$$I_p = \left(b_f^3 t/3\right) + A_s b_f^2 \tag{2}$$

$$J = \left(b_f t^3/3\right) + J_s \tag{3}$$

where for lips

$$J_s = b_L t^3/3$$

and for circular bulbs

$$J_s = \pi d^4/32$$

The approximation for J and J_s for lips introduces little error for $(b_f + b_L)/t > 5$. By assuming that the stiffened flange is fixed in the longitudinal direction at the hinge line, the major part of the torsion-bending constant C is given by

$$C = I_{y_s} b_f^2 \tag{4}$$

Furthermore, for a hinged flange the wave-length term λ in equation (1) may be replaced by the length L.

By letting $\nu = 0.3$, equation (1) becomes

$$\sigma_{cr} = \frac{E}{I_p}\left(\frac{J}{2.6} + \frac{\pi^2 C}{L^2}\right) \tag{5}$$

SOLUTION

For the idealized lip-stiffened flange shown in figure 1, the constants of equations (2), (3), and (4) become:

$$\left.\begin{aligned} I_p &= \left(b_f^3 t/3\right) + b_L t b_f^2 \\ J &= \left(b_f + b_L\right)t^3/3 \\ C &= b_L^3 t b_f^2/3 \end{aligned}\right\} \tag{6}$$

Substituting equations (6) into equation (5) gives

$$\sigma_{cr} = \left[\frac{0.388\left(1 + b_L/b_f\right) + \pi^2\left(b_L/b_f\right)^3\left(b_f/t\right)^2\left(b_f/L\right)^2}{1 + \left(3b_L/b_f\right)}\right]E\left(\frac{t}{b_f}\right)^2 \tag{7}$$

Similarly, for the idealized round bulb-stiffened flange of figure 1, the constants of equations (2), (3), and (4) become:

$$
\left.\begin{aligned}
I_p &= \left(b_f{}^3 t/3\right) + \left(\pi d^2 b_f{}^2/4\right) \\
J &= \left(b_f t^3/3\right) + \left(\pi d^4/32\right) \\
C &= \frac{\pi b_f{}^2 t^4}{8}\left[\frac{5}{8}\left(\frac{d}{t}\right)^4 - \left(\frac{d}{t}\right)^3 + \frac{1}{2}\left(\frac{d}{t}\right)^2\right]
\end{aligned}\right\} \tag{8}
$$

Substituting equations (8) into equation (5) gives

$$
\sigma_{cr} = \left\{\frac{0.388\left[\frac{b_f}{t} + 0.294\left(\frac{d}{t}\right)^4\right] + \frac{3\pi^3}{8}\left(\frac{b_f}{L}\right)^2\left[\frac{5}{8}\left(\frac{d}{t}\right)^4 - \left(\frac{d}{t}\right)^3 + \frac{1}{2}\left(\frac{d}{t}\right)^2\right]}{\left(b_f/t\right) + (3\pi/4)(d/t)^2}\right\} E\left(\frac{t}{b_f}\right)^2 \tag{9}
$$

It can be observed that both critical-stress equations contain an L/b_f term. For many calculations, such as the determination of crippling strength, it is convenient to have design charts from which the critical stress or strain may be determined directly. In order to construct such charts, however, a value of L/b_f must be prescribed.

In order to construct design charts which will have application in crippling-strength problems, a value of $L/b_f = 3.5$ was chosen. This value was obtained from the recommendations of reference 9 which are concerned with the desirable lengths of Z-sections and channel sections for local instability tests. Below $L/b_f = 3.5$, an increase in buckling and crippling stress associated with short lengths of such section occurs. Since the stiffened flange will generally be used in conjunction with other web and flange elements, this value of L/b_f appears to be reasonable.

By substituting $L/b_f = 3.5$ into equations (7) and (9), the design charts presented in figures 3 and 4 were prepared. The lower limit is that of the unstiffened flange. The upper limit is associated with lip or bulb dimensions which are sufficient to cause the stiffened flange to act as a simply supported plate or web element.

The data presented in figures 3 and 4 are for elastic torsional buckling. Since the buckling mode of the stiffened hinged flange is predominantly rotation or twisting, the secant modulus should be a good approximation to the effective modulus when buckling is inelastic. Thus, the buckling stress may be computed from

$$
\sigma_{cr} = E_s \epsilon_{cr} \tag{10}
$$

EXPERIMENTAL DATA

In reference 4, experimental data on the buckling of lipped Z-sections are presented. The proportions of the element tested were $b_f/b_w = 0.7$ and $b_w/t = 29$ and b_L/b_f was systematically varied between 0.18 and 1.00. The majority of the test points were above the proportional limit.

For the $b_L/b_f = 0.18$ data, it is quite clear that buckling of the lip-flange combination occurred in the torsional mode considered herein. For all the other data, local buckling of the web or lip occurred.

The test data on torsional instability are shown in figure 5 in conjunction with the theoretical critical strain values obtained from figure 3. It can be concluded that good agreement between theory and test data exists.

Research Division, College of Engineering,
New York University,
New York, N. Y., May 27, 1955.

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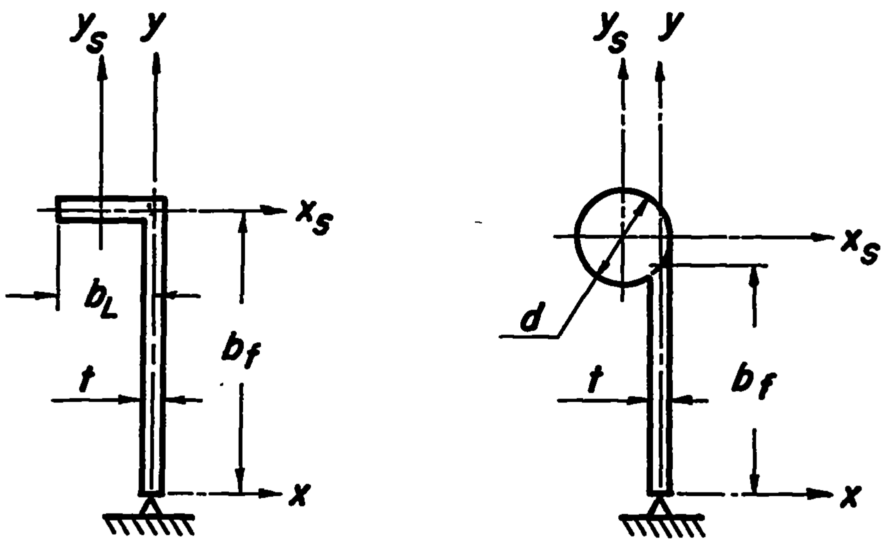

(a) Lip-stiffened flange. (b) Bulb-stiffened flange.

Figure 1.- Idealized stiffened flanges.

(a) Rotation, no distortion. (b) Distortion, no rotation.

Figure 2.- Possible buckling configurations of lip-stiffened flanges.

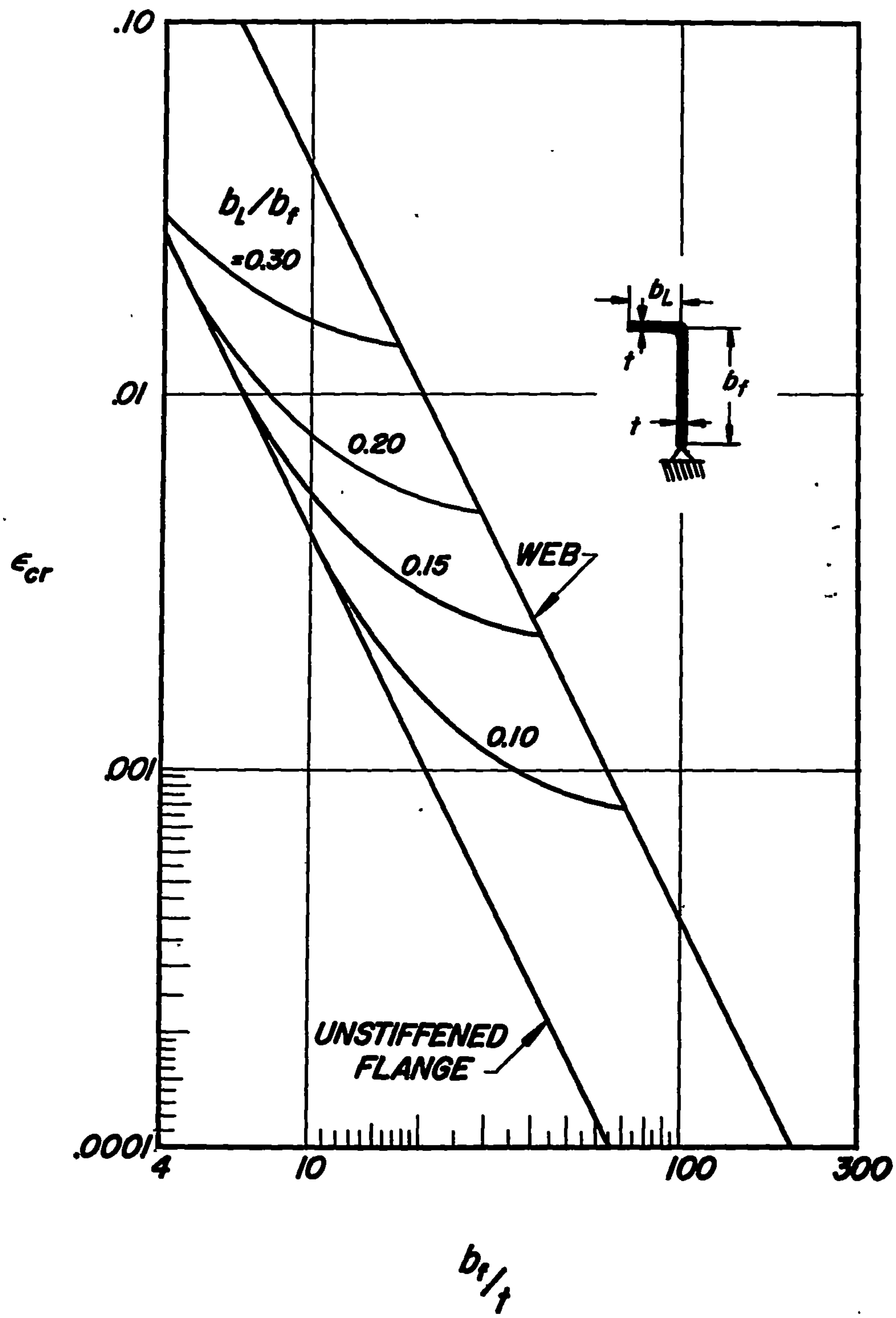

Figure 3.- Buckling strain of hinged lip flanges. $L/b_f = 3.5$; $\nu = 0.3$.

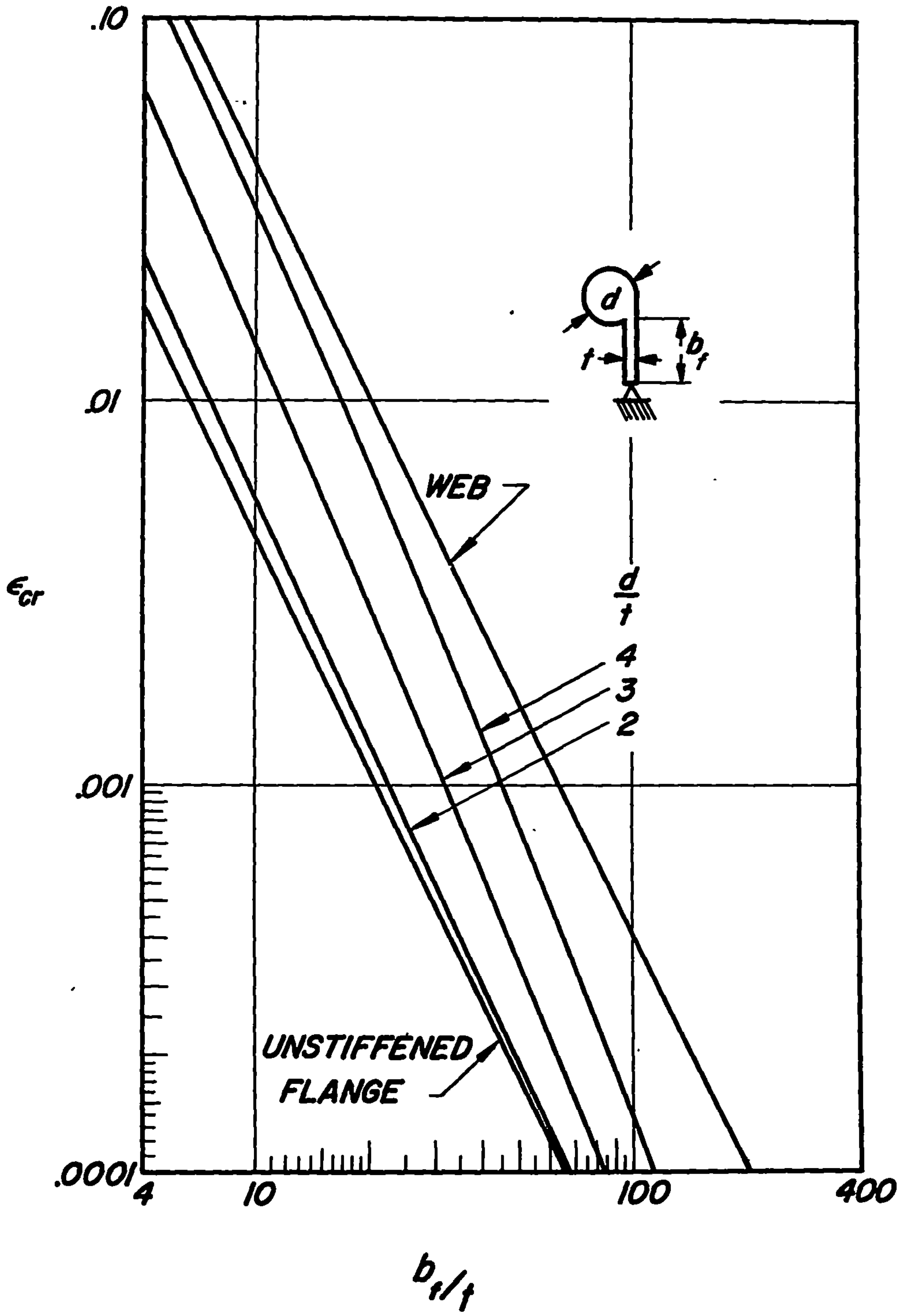

Figure 4.- Buckling strain of hinged bulb flanges. $L/b_f = 3.5$; $\nu = 0.3$.

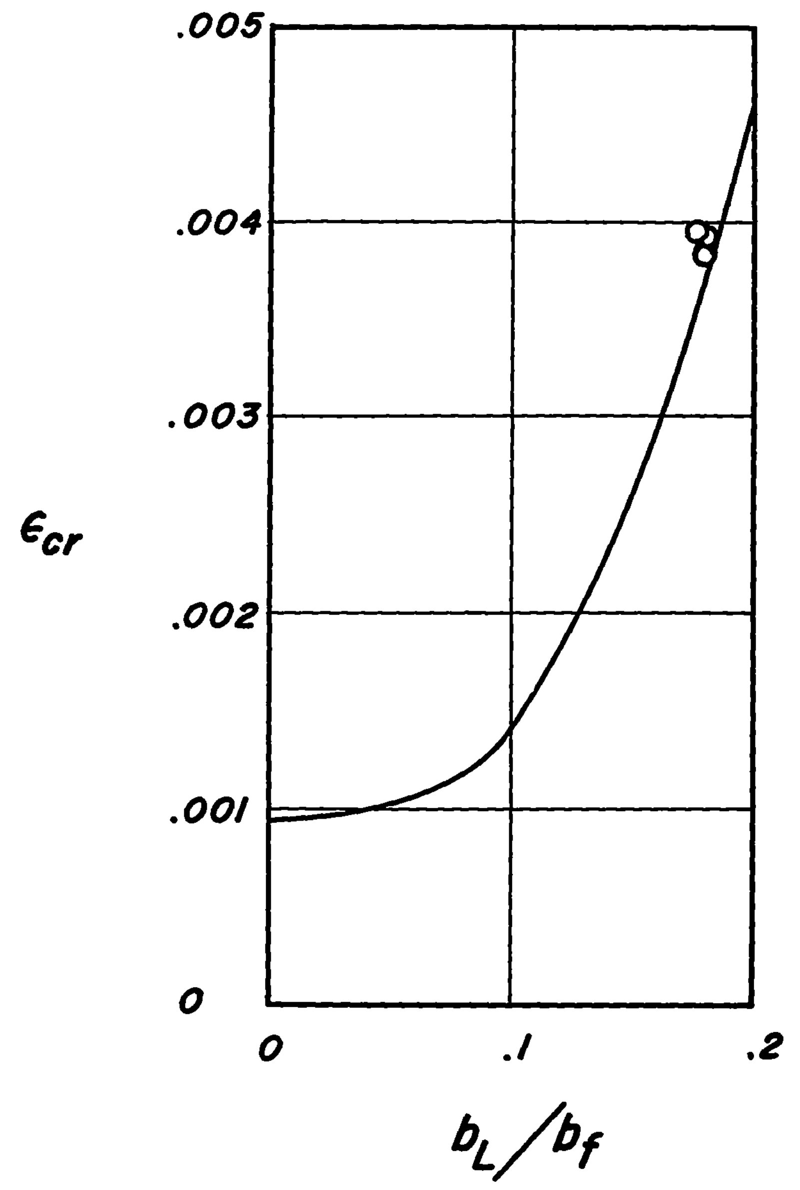

Figure 5.- Comparison of test data on lipped Z-sections and theory for lipped hinged flange. $b_f/t = 20.3$.